

Review of Application of Inelastic Response Spectra for Seismic Analysis of Structures

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Abstract—The design response spectrum has been widely used in seismic design to estimate force and deformation demands of structures imposed by earthquake ground motion. Most structures in seismic regions are designed for various seismic effects using code specified equivalent forces to account for the inertial forces generated in the event of an earthquake. These design forces are much smaller than those which would actually be generated if the structure were to remain elastic even in an earthquake of moderate intensity. Thus, the assumption that inelastic action will occur is inherent and hence the use of inelastic spectrum becomes essential for seismic analysis. Also the seismic codes require from the seismically designed structures to be capable to withstand inelastic deformations hence the development of different inelastic spectra were studied with the aim to simplify the evaluation of inelastic deformation and performance of structures. Thus, the concept of inelastic spectra needs to be adopted for performance based seismic design through capacity spectrum methods. This paper therefore aims to present the review of application of inelastic spectra in design of buildings, evaluation of peak inelastic deformation and its usage in direct displacement based design of structures from various sources.

1. INTRODUCTION

In the preliminary design of structures the maximum structural responses are of greater interest and thus it would be helpful to develop design guidelines which indicate how these peak response parameters vary with the dynamic, mechanical and damping characteristics of a structure for a given excitation. Such guidelines can be easily formulated for single degree of freedom systems in the form of inelastic response spectra.

The concept of inelastic spectra was first introduced by Veletsos and Newmark [7]. The construction of inelastic response spectra as suggested by Newmark is widely used for evaluation purposes. A multiple no. of factors influence the shape of the inelastic response spectra such as ground motions, hysteresis model etc.

Designing a structure to remain elastic under earthquake loading is impractical for most of the structures. Thus the use of inelastic response spectra becomes more rational which combines the simplicity required for eventual incorporation

into a code approach with the advantage that inelastic demands are explicitly considered.

2. INELASTIC RESPONSE SPECTRA AND ITS CONSTRUCTION

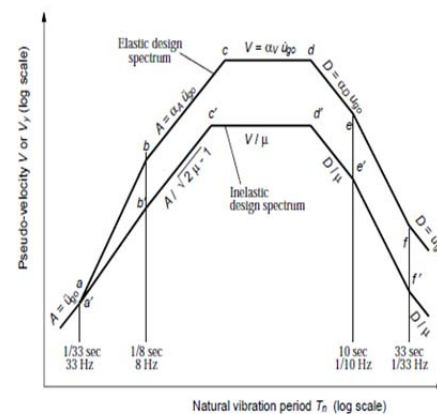


Fig. 1: Inelastic response spectra (Source: Chopra and Goel)

A simple means of representing structural response to a given earthquake motion is through a tripartite logarithmic plot response spectra. The Fig. 1 above shows a typical inelastic response spectra constructed by reducing the ordinates of the elastic spectra, on a tripartite logarithmic plot. According to Newmark, the inelastic spectrum S_{μ} can be obtained by deamplifying the elastic spectrum S_e , so that $S_{\mu} = \emptyset_{\mu} S_e$

where, \emptyset_{μ} = deamplification factor depending on the hysteresis model and the ductility factor. The value of \emptyset_{μ} as proposed by Newmark is available in a tabulated form.

The effect of damping is also considered while deriving the same.

2.1. Application of inelastic response spectra for design of R/C frames

To design a structure, an earthquake ground motion is selected and inelastic response spectra is obtained as discussed earlier.

Once the inelastic response spectra is available, design using it may proceed in the manner outlined as follows (Source: Edwin G. Burdette)

1. Select preliminary member sizes depending on drift limitations and depth-to-span ratios.
2. Determine design live load depending on the function of the structure, to be chosen from the specified building code.
3. Obtain forces and moments from dead and live loads through codal analysis.
4. Resize members as necessary based on forces and moments.
5. From a dynamic modal analysis obtain the mode shapes and periods for the frame.
6. From the inelastic response spectra plot obtain the spectral accelerations for the desired design ductility at the periods obtained earlier.
7. Apply these spectral accelerations (earthquake load), plus dead and live loads to the frame. Make a computer analysis to obtain the forces and moments at the faces of the supports.
8. Design the reinforcement using the forces and moments as obtained from the earlier step.

The above steps accomplish the use of inelastic response spectra for the design of R/C frame buildings.

2.2. Application of inelastic spectra for evaluation of peak inelastic deformation

The structure as designed in the earlier case is subjected to a pushover analysis in the computer like programme SAP2000, to obtain the Acceleration Displacement Response Spectrum (ADRS) plot as shown in Fig.2. The ADRS plot shows the plot of response spectrum and capacity spectrum obtained from push over analysis. From the ADRS plot the yield displacement (x_y) can be obtained.

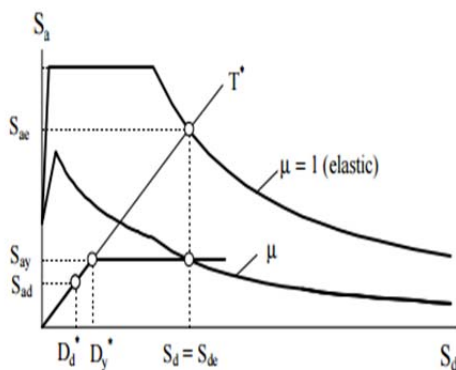


Fig. 2: ADRS plot (Source: Peter Fajfar)

The equation of motion of an inelastic single degree of freedom system when subjected to an earthquake ground motion is given as

$$mx'' + cx' + f(x, x') = -mu_g''(t) \quad (1)$$

where m , c and f represents the mass, damping and the resisting force of the system, respectively, $u_g''(t)$ denotes the earthquake acceleration. The resisting force f is defined as below

$$f = k_p x + Qz \quad (2)$$

In the above, k_p is the post yield stiffness, Q is the yield strength, and z represents the dimensionless variable that characterizes the Bounc-Wen model of hysteresis.

Substituting and simplifying eq. 1 yields

$$x'' + 2\zeta\omega x' + \alpha\omega^2 x + qgz = -u_g''(t) \quad (3)$$

in which ζ , ω , α , and q represent the damping ratio, circular frequency, post to pre yield stiffness ratio, and the yield strength coefficient (yield strength divided by system weight).

The eq. 3 is rewritten in terms of displacement ductility factor, μ . Substituting $x = \mu x_y$ we get

$$\mu'' + 2\zeta\omega\mu' + \alpha\omega^2\mu + qgz/xy = -(1/xy) u_g''(t) \quad (4)$$

the term qg/x_y in the above equation is rewritten as $\frac{qg}{xy} = \omega^2(1 - \alpha)$ (5)

Using the parameter η introduced by Mahin and Lin [4] (1983) as:

$$\eta = qg/PGA \quad (6)$$

where, PGA stands for the Peak Ground Acceleration. Incorporating η and simplifying eq. 4 we get

$$\mu'' + 2\zeta\omega\mu' + \alpha\omega^2\mu + \omega^2(1 - \alpha)z = -\omega^2(1 - \alpha) u_g''(t)/\eta \quad (7)$$

In which, $u_g''(t)$ represents the ground acceleration normalized with respect to PGA.

A ductility response spectrum is now constructed and is summarized as below:

1. Define the ground motion $u_g''(t)$.
2. Select and fix the values of ζ , α for which the spectrum is to be plotted.
3. Specify a value of η .
4. Select a value for elastic period T .
5. Determine the ductility response $\mu(t)$ of the system with T , ζ and α equal to values selected accordingly.
6. Repeat steps 4 and 5 for a range of T , resulting in the spectrum values for the η value specified in step 3.
7. Repeat steps 3 and 4 for several values of η .

The value of the ductility factor is read from the spectrum developed by the above procedure and multiplied by x_y as obtained from the ADRS plot discussed earlier to obtain the peak deformation x_m ,

$$x_m = \mu x_y \quad (8)$$

2.3. Application of inelastic spectra in direct displacement based design of structures

Direct displacement-based design is being advocated as a more rational and relevant approach to seismic design of structures compared to traditional strength-based design.

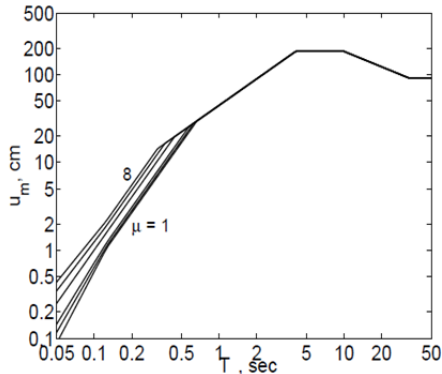


Fig. 3: Inelastic deformation spectra (Source: Chopra and Goel)

Adapted from Priestly and Calvi (1997), a direct displacement-based design procedure for SDF systems using inelastic spectra are outlined as a sequence of steps:

1. Estimate the yield deformation u_y for the system.
2. Determine the acceptable plastic rotation Θ_p of the hinge at the base.
3. Determine design displacement u_m .

$$u_m = u_y + h\Theta_p \quad (9)$$
 and design ductility factor $\mu = u_m/u_y$.
4. From Fig.3. with known u_m and μ , read T_n . Determine the initial stiffness

$$k = \left(\frac{4\pi^2}{T_n^2}\right) m \quad (10)$$
5. Determine the required yield strength

$$f_y = ku_y \quad (11)$$
6. Estimate member sizes and detailing to provide the strength determined by above eq. For the resulting design of the structure, calculate the initial stiffness and yield deformation $u_y = f_y/k$.
7. Obtain, $\Theta_p = (u_m - u_y)/h$. (12)
8. Repeat steps 3 to 6 for the convergence of $\Theta_{p(\text{calculated})}$ (eq. 12) and $\Theta_{p(\text{assumed})}$ (eq. 9).

These steps satisfactorily accomplish the application of inelastic response spectra.

3. ILLUSTRATIVE EXAMPLE

In this paper only one design example is illustrated to show the application of the inelastic response spectra because of space limitations [1,2]. The design example considered shows the use of inelastic spectra in direct displacement based design of a bridge pier.

Height of bridge pier, $h = 9\text{m}$.

Take $k = \frac{3EI}{h^3}$

Estimating $u_y = 4.5\text{cm}$.

Assuming $\Theta_p = 0.02 \text{ rad}$.

From eq. 9 $u_m = u_y + h\Theta_p = 4.5 + 900 \times 0.02 = 22.5\text{cm}$ and $\mu = u_m/u_y = 22.5/4.5 = 5$.

The deformation design spectrum for inelastic system is shown in Fig.4. Corresponding to $u_m = 22.5\text{cm}$, this spectrum gives $T_n = 1.01\text{s}$ and k is computed by eq. 10, $k = 298.7 \text{ kN/cm}$. Yield strength is calculated by eq. 11, $f_y = 1344\text{kN}$. The circular column is then designed using ACI318-95 for axial load due to superstructure weight of 7517 kN plus column self weight of 375 kN and the bending moment due to lateral force $= f_y \cdot M = hf_y = 12096 \text{ kN-m}$. For the resulting column design, $\rho_t = 3.62\%$ flexural strength $= 12976 \text{ kN-m}$, and lateral strength $= 1441 \text{ kN}$. For $\rho_t = 3.62\%$, $EI = 4.24 \times 10^6 \text{ kN - m}^2$ (ACI 318-95); using this EI gives $k = 174.4 \text{ kN/cm}$. The yield deformation is $u_y = f_y/k = 1441/174.4 = 8.27 \text{ cm}$. since the yield deformation computed differs significantly from the initial estimate of $u_y = 4.5\text{cm}$, iterations are necessary. The results of such iterations are summarized in table 1. The procedure converged after five iterations giving a column design with $\rho_t = 5.5\%$. The column has an initial stiffness, $k = 238.6 \text{ kN/cm}$ and lateral yield strength, $f_y = 1907 \text{ kN}$.

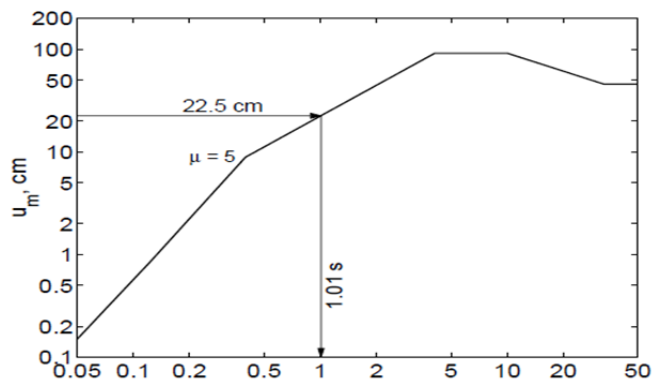


Fig. 4. Calculation of T_n using inelastic deformation spectra (Source: Chopra and Goel)

Table 1: Iterations of the direct displacement based design

No.	u_y (cm)	u_m (cm)	μ	T_n (s)	k (kN/cm)	f_y (kN)	ρ_t (%)	Design f_y (kN)	Design k (kN/cm)	u_y (cm)
1	4.50	22.5	5.00	1.01	298.7	1344	3.62	1441	174.4	8.27
2	8.27	26.3	3.18	1.18	219.1	1812	5.55	1912	240.3	7.96
3	7.96	26.0	3.26	1.16	224.4	1786	5.43	1899	236.2	8.04
4	8.04	26.0	3.24	1.17	223.0	1793	5.50	1907	238.6	7.99
5	7.99	26.0	3.25	1.16	223.8	1789	5.50	1907	238.6	7.99

3.1. Discussion

The plastic rotation Θ_p is calculated using eq. 12 and is obtained as 0.0199 rad. It is essentially identical to the acceptable value of 0.02 rad, imposed on the design. Clearly the procedure has produced a satisfactory design.

4. CONCLUSION

The use of inelastic response spectra has been exclusively reviewed in this paper in accordance with the three cases shown above. However the applicability of the inelastic spectra is influenced by a large number of parameters, hence they should be considered explicitly during the usage. The inelastic spectra also provide a comparative study of various analysis techniques in seismic analysis.

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